

Marwari college Darbhanga

Subject---physics (Hons)

Class--- B. Sc. Part 1

Paper---02 ; group----A

Topic--- Carnot's Heat Engine( Thermal physics )

Lecture series- 29

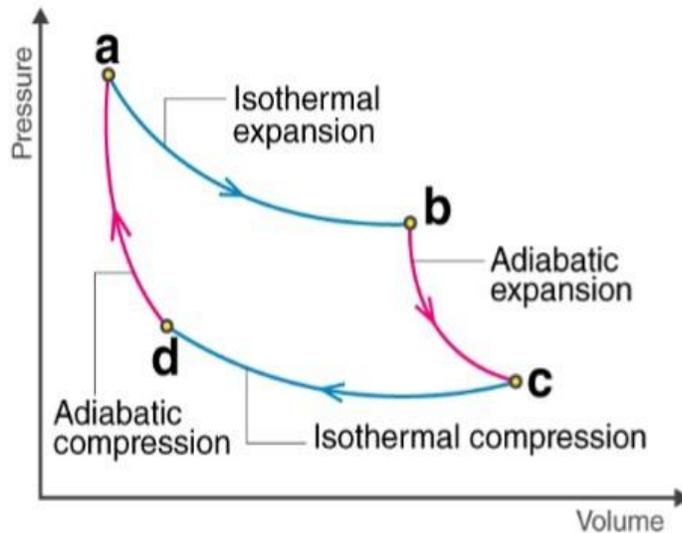
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### Carnot's Heat Engine

Carnot engine is a theoretical thermodynamic cycle proposed by **Leonard Carnot**. It gives the estimate of the maximum possible efficiency that a heat engine during the conversion process of heat into work and conversely, working between two reservoirs, can possess.



**Carnot's theorem states that:**

- Heat engines that are working between two heat reservoirs are less efficient than the Carnot heat engine that are operating between same reservoirs.
- Irrespective of the operation details, every Carnot engine is efficient between two heat reservoirs.
- Maximum efficiency is given as:

$$\eta_{\max} = \eta_{\text{Carnot}} = 1 - T_c / T_h$$

Where,

$T_c$ : absolute temperature of cold reservoir

$T_h$ : absolute temperature of hot reservoir

$\eta$ : ratio of work done by the engine to heat drawn out of the hot reservoir

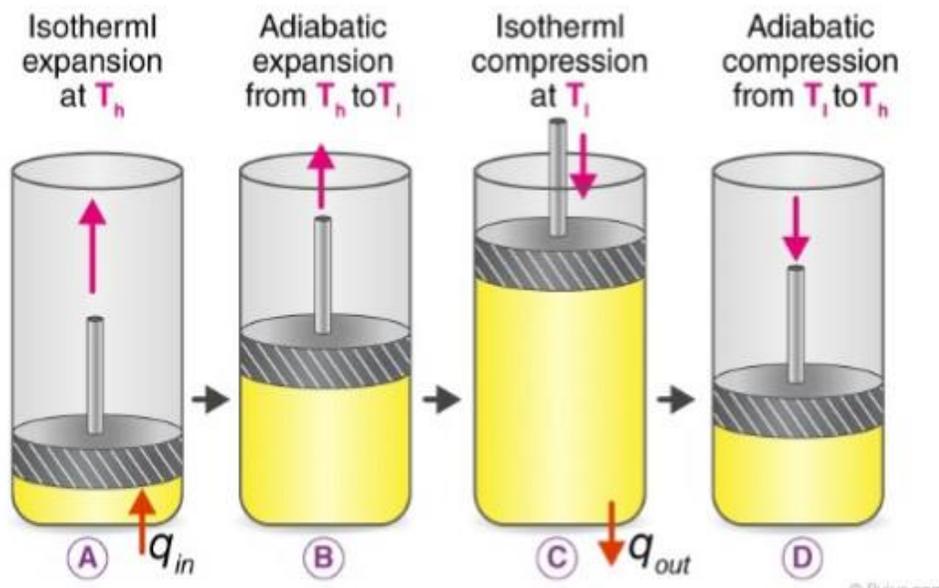
wo given temperatures  $T_1$  (hot reservoir) and  $T_2$  (cold reservoir), can never have an efficiency more than the

Carnot engine working between the same reservoirs respectively.

Also, the efficiency of this type of engine is independent of the nature of the working substance and is only dependent on the temperature of the hot and cold reservoirs.

### Carnot Cycle:

A Carnot cycle is defined as an ideal reversible closed thermodynamic cycle in which there are four successive operations involved and they are isothermal expansion, adiabatic expansion, isothermal compression, and adiabatic compression. During these operations, the expansion and compression of substance can be done up to desired point and back to initial state.



Following are the four processes of Carnot cycle:

- In (a), the process is reversible isothermal gas expansion. In this process, the amount of heat absorbed by the ideal gas is  $q_{in}$  from the heat source which is at a temperature of  $T_h$ . The gas expands and does work on the surroundings.
- In (b), the process is reversible adiabatic gas expansion. Here, the system is thermally insulated and the gas continues to expand and work is done on the surroundings. Now the temperature is lower,  $T_l$ .
- In (c), the process is reversible isothermal gas compression process. Here, the heat loss,  $q_{out}$  occurs when the surroundings do the work at temperature  $T_l$ .
- In (d), the process is reversible adiabatic gas compression. Again the system is thermally insulated. The temperature again rise back to  $T_h$  as the surrounding continue to do their work on the gas.

*For an ideal gas operating inside a Carnot cycle, the following are the steps involved:*

### **Step 1:**

Isothermal expansion: The gas is taken from  $P_1, V_1, T_1$  to  $P_2, V_2, T_2$ . Heat  $Q_1$  is absorbed from the reservoir at temperature  $T_1$ . Since the expansion is isothermal, the total change in internal energy is zero and the heat absorbed by the gas is equal to the work done by the gas on the environment, which is given as:

$$W_{1 \rightarrow 2} = Q_1 = \mu \times R \times T_1 \times \ln v_2 / v_1$$

### **Step 2:**

Adiabatic expansion: The gas expands adiabatically from  $P_2, V_2, T_1$  to  $P_3, V_3, T_2$ .

Here work done by the gas is given by:

$$W_{2 \rightarrow 3} = \frac{\mu R}{\gamma - 1} (T_1 - T_2)$$

**Step 3:**

Isothermal compression: The gas is compressed isothermally from the state  $(P_3, V_3, T_2)$  to  $(P_4, V_4, T_2)$ .

Here, the work done on the gas by the environment is given by:

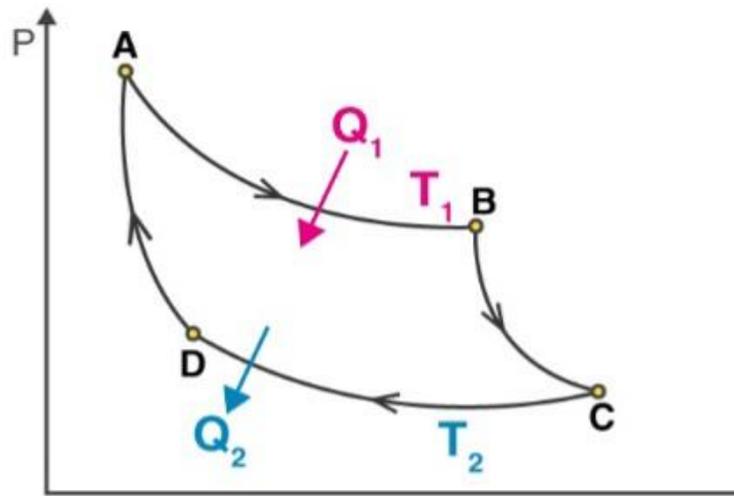
$$W_{3 \rightarrow 4} = \mu R T_2 \ln \frac{v_3}{v_4}$$

**Step 3:**

Isothermal compression: The gas is compressed isothermally from the state  $(P_3, V_3, T_2)$  to  $(P_4, V_4, T_2)$ .

Here, the work done on the gas by the environment is given by:

$$W_{4 \rightarrow 1} = \frac{\mu R}{\gamma - 1} (T_1 - T_2)$$



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Hence, the total work done by the gas on the environment in one complete cycle is given by:

$$\begin{aligned} W &= W_{1 \rightarrow 2} + W_{2 \rightarrow 3} + W_{3 \rightarrow 4} \\ &+ W_{4 \rightarrow 1} \\ W &= \mu R T_1 \ln \frac{v_2}{v_1} - \mu R T_2 \ln \frac{v_3}{v_4} \end{aligned}$$

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*Net efficiency*

$$= \frac{\text{Net workdone by the gas}}{\text{Heat absorbed by the gas}}$$

$$\begin{aligned} \text{Net efficiency} &= \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} \\ &= 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1} \frac{\ln \frac{v_3}{v_4}}{\ln \frac{v_2}{v_1}} \end{aligned}$$

Since the step 2→3 is an adiabatic process, we can write  $T_1 V_2^{\gamma-1} = T_2 V_3^{\gamma-1}$

Or,

$$\frac{v_2}{v_3} = \left( \frac{T_2}{T_1} \right)^{\frac{1}{\gamma-1}}$$

Similarly, for the process 4→1, we can write

$$\frac{v_1}{v_2} = \left( \frac{T_2}{T_1} \right)^{\frac{1}{\gamma-1}}$$

This implies,

$$\frac{v_2}{v_3} = \frac{v_1}{v_2}$$

**So, the expression for net efficiency of carnot engine reduces to:**

$$\text{Net efficiency} = 1 - \frac{T_2}{T_1}$$